nag_binomial_dist (g01bjc)

1. Purpose

nag_binomial_dist (g01bjc) returns the lower tail, upper tail and point probabilities associated with a Binomial distribution.

2. Specification

```
#include <nag.h>
#include <nagg01.h>

void nag_binomial_dist(Integer n, double p, Integer k, double *plek,
         double *pgtk, double *peqk, NagError *fail)
```

3. Description

Let X denote a random variable having a Binomial distribution with parameters n and p ($n \geq 0$ and $0 < p < 1$). Then

$$\text{Prob}\{X = k\} = \binom{n}{k} p^k (1-p)^{n-k}, \quad \text{for} \quad k = 0, 1, .., n.$$

The mean of the distribution is np and the variance is $np(1-p)$.

This routine computes for given n, p and k the probabilities:

plek $= \text{Prob}\{X \leq k\}$
pgtk $= \text{Prob}\{X > k\}$
peqk $= \text{Prob}\{X = k\}$.

The method is similar to the method for the Poisson distribution described in Knüsel (1986).

4. Parameters

n
Input: the parameter n of the Binomial distribution.
Constraint: $\mathbf{n} \geq 0$.

p
Input: the parameter p of the Binomial distribution.
Constraint: $0.0 < \mathbf{p} < 1.0$.

k
Input: the integer k which defines the required probabilities.
Constraint: $0 \leq \mathbf{k} \leq \mathbf{n}$.

plek
Output: the lower tail probability, $\text{Prob}\{X \leq k\}$.

pgtk
Output: the upper tail probability, $\text{Prob}\{X > k\}$.

peqk
Output: the point probability, $\text{Prob}\{X = k\}$.

fail
The NAG error parameter, see the Essential Introduction to the NAG C Library.

5. Error Indications and Warnings

NE_INT_ARG_LT
On entry, **n** must not be less than 0: $\mathbf{n} = \langle value \rangle$.
On entry, **k** must not be less than 0: $\mathbf{k} = \langle value \rangle$.

NE_2_INT_ARG_GT
On entry, $\mathbf{k} = \langle value \rangle$ while $\mathbf{n} = \langle value \rangle$. These parameters must satisfy $\mathbf{k} \leq \mathbf{n}$.

NE_ARG_TOO_LARGE
On entry, **n** is too large to be represented exactly as a double precision number.

NE_REAL_ARG_LE
On entry, **p** must not be less than or equal to 0.0: $\mathbf{p} = \langle value \rangle$.

NE_REAL_ARG_GE
On entry, **p** must not be greater than or equal to 1.0: $\mathbf{p} = \langle value \rangle$.

NE_VARIANCE_TOO_LARGE
On entry, the variance ($= np(1-p)$) exceeds 10^6.

NE_INTERNAL_ERROR
An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please consult NAG for assistance.

6. Further Comments

The time taken by the routine depends on the variance ($= np(1-p)$) and on k. For given variance, the time is greatest when $k \approx np$ (= the mean), and is then approximately proportional to the square-root of the variance.

6.1. Accuracy

Results are correct to a relative accuracy of at least 10^{-6} on machines with a precision of 9 or more decimal digits, and to a relative accuracy of at least 10^{-3} on machines of lower precision (provided that the results do not underflow to zero).

6.2. References

Knüsel L (1986) Computation of the Chi-square and Poisson Distribution. *SIAM J. Sci. Statist. Comput.* **7** 1022–1036.

7. See Also

nag_poisson_dist (g01bkc)
nag_hypergeom_dist (g01blc)

8. Example

This example program reads values of n and p from a data file until end-of-file is reached, and prints the corresponding probabilities.

8.1. Program Text

```
/* nag_binomial_dist(g01bjc) Example Program.
 *
 * Copyright 1996 Numerical Algorithms Group.
 *
 * Mark 4, 1996.
 *
 */

#include <nag.h>
#include <nag_stdlib.h>
#include <stdio.h>
#include <nagg01.h>

main()
{

  double plek, peqk, pgtk;
  double p;

  Integer k, n;
```

```
  Vprintf("g01bjc Example Program Results\n");
  /*     Skip heading in data file */
  Vscanf("%*[^\n] ");

  Vprintf("\n");
  Vprintf("   n     p      k     plek      pgtk      peqk\n\n");

  while ((scanf("%ld %lf %ld%*[^\n]", &n, &p, &k)) != EOF)
    {
      g01bjc(n, p, k, &plek, &pgtk, &peqk, NAGERR_DEFAULT);
      Vprintf("%5ld%8.3f%5ld%10.5f%10.5f%10.5f\n",n, p, k, plek, pgtk, peqk);
    }

  exit(EXIT_SUCCESS);
}
```

8.2. Program Data

```
g01bjc Example Program Data
4  0.50    2   : n, p, k
19  0.44   13
100  0.75   67
2000  0.33  700
```

8.3. Program Results

```
g01bjc Example Program Results

   n     p      k     plek      pgtk      peqk

    4   0.500    2   0.68750   0.31250   0.37500
   19   0.440   13   0.99138   0.00862   0.01939
  100   0.750   67   0.04460   0.95540   0.01700
 2000   0.330  700   0.97251   0.02749   0.00312
```